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Key Points:

- Two barotropic instabilities facilitate a meso- γ -scale convective storm leading to a record-breaking rainfall
- The environment feeds energy to the convective storm in the lower layer while extracts energy from it in the upper layer
- The multiscale window transform-based multiscale energetics is extended to an atmosphere with moisture

Correspondence to:

X. S. Liang,
xsliang@fudan.edu.cn

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Author Contributions:

Conceptualization: Jiwang Ma

Formal analysis: Jiwang Ma

Funding acquisition: X. San Liang

Investigation: Jiwang Ma, X. San Liang

Methodology: Jiwang Ma, Yineng Rong, X. San Liang

Software: Jiwang Ma, Yineng Rong

Supervision: X. San Liang, Renhe Zhang, Xu Tang

Validation: Jiwang Ma

Writing – original draft: Jiwang Ma, X. San Liang

Writing – review & editing: Jiwang Ma, X. San Liang, Renhe Zhang, Xu Tang

Moist Available Potential Energy, Multiscale Energetics and the Multiscale Dynamics Governing an Extremely Devastating Rainstorm

Jiwang Ma^{1,2} , Yineng Rong^{1,3}, X. San Liang^{1,3} , Renhe Zhang^{1,3} , and Xu Tang^{1,3}

¹Department of Atmospheric and Oceanic Sciences, and Institute of Atmospheric Sciences, Fudan University, Shanghai, China, ²The Artificial Intelligence Department, Division of Frontier Research, Southern Marine Science and Engineering Guangdong Laboratory (Zhuhai), Zhuhai, China, ³IRDR ICoE on Risk Interconnectivity and Governance on Weather/Climate Extremes Impact and Public Health, Fudan University, Shanghai, China

Abstract An unprecedented record-breaking extreme hourly rainfall event (201.9 mm/hr) occurred on 20 July 2021, at the Zhengzhou Station, China. In previous studies, a meso- γ -scale convective storm is found to be the direct process causing the rainfall; also found is the impact of processes on other scales (synoptic- and meso-scales), in enhancing moisture supplement, wind convergence, vertical wind shear, etc. How these processes, which lie on distinct scales, may interact, and/or, how the interaction may play a role, remain elusive. Motivated by the seminal theoretical work of Lilly (1986, [https://doi.org/10.1175/1520-0469\(1986\)043<0113:tseapo>2.0.co;2](https://doi.org/10.1175/1520-0469(1986)043<0113:tseapo>2.0.co;2)), we hence explore this aspect of the storm dynamics, using the functional analysis tool, multiscale window transform (MWT), and the theory of canonical transfer. We first extend the MWT-based multiscale energetics to an atmosphere with moisture so as to deal with processes like rainstorm, then apply MWT to reconstruct the fields onto three scale windows, namely, the synoptic-scale, meso- β -scale, and meso- γ -scale windows. The reconstructed meso- γ -scale process is shown to characterize the storm. Contribution to its growth from other scales is hence measured by the canonical energy transfers from the ambient. It is found that, beneath 700 hPa, locally the system is dominated by a primary barotropic instability and a secondary barotropic instability: both the synoptic and meso- β -scale processes fuel the storm through canonical kinetic energy (KE) transfers. But above 700 hPa, the transfer direction is reversed, forming a KE sink. By calculation, the anisotropy of the storm wind accounts for the reversal. In comparison to previous studies, we highlight the importance of the energy extracted from the ambient through barotropic instabilities, which contribute directly to the record-breaking rainfall in the lower layer.

Plain Language Summary A record-breaking extreme hourly rainstorm occurred on 21 July 2021 in Zhengzhou, China, resulting in heavy casualties, severe infrastructure damages, and economic losses. During the rainstorm, 201.9 mm of precipitation poured down in just 1 hr, which is almost one third of the total yearly climatological precipitation at the same station. This study attempts to supply some understanding of its underlying dynamics from a multiscale point of view. It is found that the triggering of the highly concentrated rainfall is through a primary barotropic instability and a secondary barotropic instability in the lower layer, but in the upper layer, the multiscale interaction forms a sink for the kinetic energy gain, making a cycle in the vertical direction. This study adds on the existing literature new knowledge from a perspective of localized energetics, which is expected to help improve the numerical models toward better operational forecasts.

1. Introduction

From 17 July through 21 July 2021, an unprecedented rainstorm occurred over Zhengzhou, China, causing a severe flooding and leaving 292 people dead, 47 people missing, and a direct economic loss in the amount of nearly 53.2 billion RMB (e.g., Su et al., 2021). Particularly, the maximum hourly rainfall accumulated from 1600 through 1700 BJT (Beijing Time) at the Zhengzhou station reached a record-breaking 201.9 mm (see Figures 1 and 2), while the climatologically averaged yearly rainfall at the station is only 641 mm (e.g., Su et al., 2021). In other words, almost one third of the rainfall of the whole year poured down at this station in just 1 hr. Unfortunately, all the state-of-the-art operational numerical models underestimated this devastating extreme rainfall event to a large extent. The predicted maximum of the daily rainfall was only about 130–150 mm (Chyi et al., 2022), which was even less than the above-mentioned hourly rainfall, let alone the daily rainfall (it was

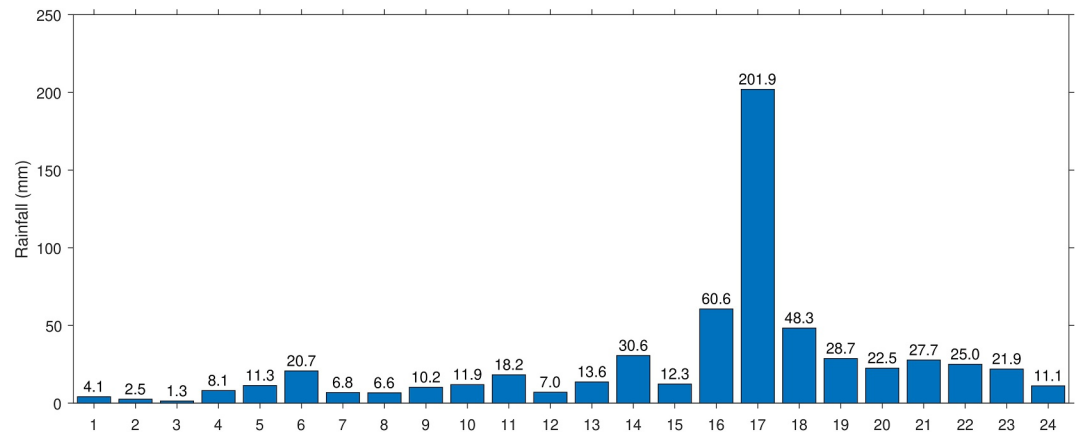


Figure 1. Time series of the hourly rainfall (units: mm) at the Zhengzhou station during 00:00–24:00 BJT (Beijing Time), 20 July 2021.

actually more than 600 mm, i.e., almost the whole year's rainfall; see Su et al., 2021). As a result, the model forecasts did not provide enough information to issue in-advance warning of the danger of an extreme rainstorm. For this reason, it is of great scientific and societal importance to investigate the underlying dynamical processes that lead to such a devastating event, the extreme hourly rainfall in particular.

As demonstrated in recent studies, a meso- γ -scale convective cell is found to be the direct process leading to the record-breaking hourly rainfall. In its development and maintenance, this cell involves processes on distinct scales (e.g., Li et al., 2025; Sun et al., 2023; Wei et al., 2023; Yin, Gu, Liang, et al., 2022). To be specific, the large-scale circulations, that is, a typhoon and the Western Pacific Subtropical High on its north side, set up a northwestward wind corridor in the lower troposphere, transporting moisture from the ocean to the ambient area where the convective system occurred (e.g., Xu et al., 2022). Several mesoscale processes are also believed to be substantial to the convective cell, including a mesoscale cyclone (e.g., Sun et al., 2023; Wei et al., 2023; Yin, Gu, Liang, et al., 2022), a barrier jet (e.g., Sun et al., 2023; Wei et al., 2023), and a downslope gravity current (Sun et al., 2023). These mesoscale processes provide a favorable environment for the convective cell, through enhancing the vertical

wind shear, inducing wind convergence, changing the stability of the atmosphere, etc. For the meso- γ -scale convection per se, its dynamics has been discussed from the perspective of vertical momentum equation under anelastic approximation (Li et al., 2025). In their study, buoyancy and dynamic acceleration are both found to be essential for the formation and maintenance of the updraft of the convective cell.

Though the impacts of the processes on distinct scales on the meso- γ -scale convection have been discussed in previous studies, how these scales may interact, and how the interaction, which is characterized by the cross-scale exchange of energy, may play a role in the cell's development and maintenance is still an open question. Lilly (1986) proposes that the meso- γ -scale storm can extract a substantial fraction of kinetic energy (KE) from the mean flow. A natural question is whether the energy exchange contributes to the extreme hourly rainfall. We henceforth will focus on this exchange between the convective cell and the circulations on other scales. To be specific, this study will address a new perspective on the dynamics of the convections, that is, the interactions among scales and their roles in the development and maintenance of the cell. Questions to be asked include, for example, whether it fuels other scale dynamical processes or gains energy from them, what kind of processes control the interactions (baroclinic instability, barotropic instability, or both), and so forth. Out of these concerns, this study will be conducted in a multiscale framework. Particularly, a faithful representation of

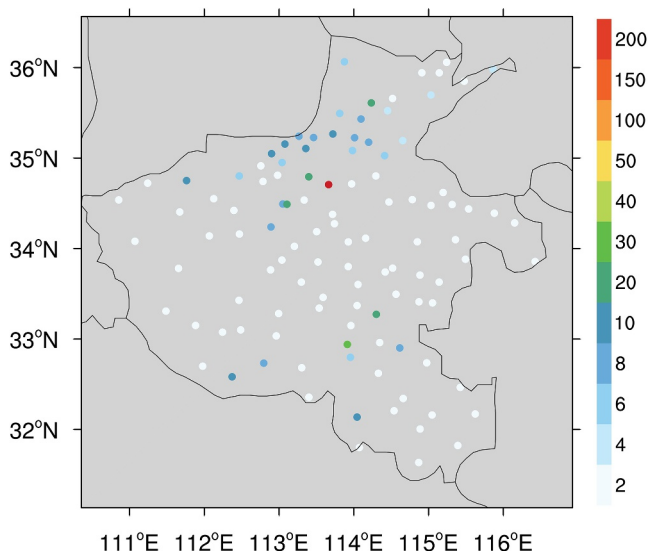


Figure 2. Spatial distribution of the hourly rainfall (units: mm) during 16:00–17:00 BJT, 20 July 2021.

energy transfer among scales will be emphasized because of its key role played in the storm; this is made possible by the “canonical transfer” theory established by Liang (2016) and Liang and Robinson (2005, 2007) (refer to Section 2 for details).

This paper is organized as follows: The following is an introduction of the key methodology for our analysis, multiscale window transform and multiscale energetics analysis. Added to the original material in Liang (2016) is the multiscale available potential energetics for moist atmospheres. Section 3 introduces the data sets used for this study. Section 4 is a presentation of the model simulation of the rainstorm. Section 5 presents the results, including a description of the synoptic features and the reconstructed multiscale fields during the rainstorm event, as well as the rainstorm dynamics. Section 6 concludes the study, together with a brief discussion.

2. Multiscale Window Transform and Multiscale Energetics Analysis for Moist Atmospheres

2.1. Multiscale Window Transform

The methodology and theory used in this study include the multiscale window transform (Liang & Anderson, 2007), or MWT for short, and the MWT-based theory of canonical transfer (Liang, 2016). Only a brief introduction is given here. Those who are interested are referred to Liang and Anderson (2007) and Liang (2016) for details.

MWT is a functional analysis tool which can be used to orthogonally decompose a function space by scale into a sum of subspace called *scale windows*, or simply *windows*, while retaining the local features of the functions. In other words, this method preserves locality and orthogonality simultaneously. This property distinguishes MWT from other commonly used filtering methods. For example, Fourier decomposition is orthogonal but lacks locality, whereas Lanczos filtering retains local features but does not ensure orthogonality. Most importantly, it provides *transform coefficients*, as well filtered fields, to ensure a faithful representation of energetics in phase space through the Parseval identity (see below). Specifically, for a time series $u(t)$, consider a three-scale window decomposition, u can be expressed as a sum of three parts called *reconstructions* (or filtered parts):

$$u(t) = \sum_{\varpi=0}^2 u^{\sim\varpi}(t)$$

where $\varpi = 0, 1$ and 2 respectively represent the three windows. The reconstruction of u on window ϖ , $u^{\sim\varpi}(t)$ is

$$u^{\sim\varpi}(t) = \sum_{n=0}^{2^{j_2}-1} \hat{u}_n^{\sim\varpi} \phi_n^{j_2}(t), \quad (1)$$

where

$$\hat{u}_n^{\sim\varpi} = \int_0^1 u^{\sim\varpi}(t) \phi_n^{j_2}(t) dt, \quad (2)$$

and $\phi(t)$ denotes a localized scaling basis, with j_2 the highest scale level, and n the discrete time step in the sampling space. Equations 1 and 2 form a transform-reconstruction pair (multiscale window transform, MWT vs. multiscale window reconstruction, MWR). For each MWR of a signal $u(t)$, $u^{\sim\varpi}(t)$, there is a corresponding transform coefficient $\hat{u}_n^{\sim\varpi}$. Thanks to the Parseval relation in functional analysis, the *time-dependent* energy on window ϖ can be expressed as the square of the transform coefficients, that is, $(\hat{u}_n^{\sim\varpi})^2$ (up to some constant; cf. Liang & Anderson, 2007). Note that it is by NO means the square of the reconstructed/filtered field, that is, $[u^{\sim\varpi}(t)]$, as usually seen in the literature.

2.2. Multiscale Energetics Equations With a Moist Atmosphere

2.2.1. Available Potential Energy With Moisture Considered

In Lorenz's (1955) seminal work, the available potential energy (APE) in a dry atmosphere was defined and the multiscale energy equations are accordingly constructed, where the effect of moisture is taken as an external

forcing referred to as “latent heating.” Obviously, this would lead to a lack of moisture effect in discussing the internal dynamical processes, for example, the baroclinic instability. However, when discussing an event like the rainstorm in this paper, it may be problematic, due to the well-known important roles of moisture in such a precipitation process. For example, a moist air particle in the lower layer will gain more vertical velocity during its rising processes, once above the condensation level, than a dry one. Consequently, the effect of moisture needs to be included in the available potential energy.

We must clarify that we do not aim to solve the moist APE problem in this paper. Actually, this is an active research field with the involvement of many studies, such as Lorenz (1978), Pauluis (2007), Marsico et al. (2019), to name a few. The only purpose here is to include the moisture effect into the Lorenz energy cycle, while ignoring other processes.

The equations governing the evolutions of temperature T^* and specific humidity q_v^* are

$$\frac{\partial T^*}{\partial t} + \mathbf{v}_h \cdot \nabla_h T^* + \omega \frac{\partial T^*}{\partial p} - \frac{\omega \alpha^*}{c_p} = \frac{Q_{\text{net}}}{c_p}, \quad (3)$$

$$\frac{\partial q_v^*}{\partial t} + \mathbf{v}_h \cdot \nabla_h q_v^* + \omega \frac{\partial q_v^*}{\partial p} = r_e - r_c, \quad (4)$$

where r_e and r_c are respectively the rates of evaporation and condensation, Q_{net} stands for the heating rate from all diabatic sources, $\omega = \frac{dp}{dt}$, and the starred variables mean the whole fields, with the corresponding non-starred ones reserved for their anomalies (departures from their mean vertical profiles; see below). The subscript h indicates the component on the p plane; for example, $\mathbf{v} = (\mathbf{v}_h, \omega)$, $\nabla = (\nabla_h, \frac{\partial}{\partial p})$, and so forth. The other symbols are conventional (refer to appendix A).

Following Liang (2016), we remove the mean vertical profile from the fields of humidity, temperature, specific volume, enthalpy, etc.

$$\begin{aligned} q_v^* &= \overline{q_v}(p) + q_v(\lambda, \varphi, p; t), \\ T^* &= \overline{T}(p) + T(\lambda, \varphi, p; t), \\ \alpha^* &= \overline{\alpha}(p) + \alpha(\lambda, \varphi, p; t). \end{aligned}$$

Here the mean (denoted by overbar) is over time and the whole p -plane. Hence the equations become

$$\frac{\partial q_v}{\partial t} + \mathbf{v}_h \cdot \nabla_h q_v + \omega \frac{\partial q_v}{\partial p} + \omega \frac{\partial \overline{q_v}}{\partial p} = r_e - r_c = -\frac{Q_{cn}}{\ell_v}, \quad (5)$$

$$\frac{\partial T}{\partial t} + \mathbf{v}_h \cdot \nabla_h T + \omega \frac{\partial T}{\partial p} + \omega \left[\frac{\partial \overline{T}}{\partial p} - \frac{\overline{\alpha}}{c_p} \right] - \frac{\omega \alpha}{c_p} = \frac{Q_{\text{net}}}{c_p}, \quad (6)$$

where Q_{cn} is the latent heat release through phase change. (Here processes other than evaporation and condensation are neglected; see, e.g., Li et al., 2002). Note that $Q_{\text{net}} = Q_R + \ell_v(r_c - r_e) = Q_R + Q_{cn}$, with Q_R being the heating due to shortwave and longwave radiation.

The enthalpy anomaly, that is, the enthalpy with its time and horizontal mean removed, is

$$H = c_p T + \ell_v q_v$$

Let

$$h = \frac{H}{c_p} = T + \frac{\ell_v}{c_p} q_v,$$

where ℓ_v is the specific latent heat of vapourization (in $J/kg/K$). Equation 5 $\times \ell_v/c_p$ plus Equation 6 thus yields

$$\frac{\partial h}{\partial t} + \mathbf{v}_h \cdot \nabla_h h + \omega \frac{\partial h}{\partial p} + \omega \left[\frac{\partial \bar{q}_v \ell_v / c_p}{\partial p} + \frac{\partial \bar{T}}{\partial p} - \frac{\bar{\alpha}}{c_p} \right] - \frac{\omega \alpha}{c_p} = \frac{Q_R}{c_p}. \quad (7)$$

Multiply by h to get

$$\frac{\partial h^2/2}{\partial t} + \mathbf{v}_h \cdot \nabla_h h^2/2 + \omega \frac{\partial h^2/2}{\partial p} + \omega h \left[\frac{\partial \bar{q}_v \ell_v / c_p}{\partial p} + \frac{\partial \bar{T}}{\partial p} - \frac{\bar{\alpha}}{c_p} \right] - \frac{h \omega \alpha}{c_p} = \frac{h Q_R}{c_p}.$$

The ideal gas relation implies $\alpha^* = T^* R/p$ and hence $\bar{\alpha} = \bar{T} R/p$, $\alpha = TR/p$. So the fourth term becomes

$$\begin{aligned} h \omega \left[\frac{\partial \bar{q}_v \ell_v / c_p}{\partial p} \right] + (T + q_v \ell_v / c_p) \omega \left[\frac{\partial \bar{T}}{\partial p} - \frac{\bar{\alpha}}{c_p} \right] \\ = \frac{\ell_v}{c_p} h \omega \frac{\partial \bar{q}_v}{\partial p} - \left(\alpha \omega + q_v \frac{\ell_v}{c_p} \omega \frac{R}{p} \right) \frac{1}{c}, \end{aligned}$$

where

$$c = - \left[\frac{p}{R} \frac{\partial \bar{T}}{\partial p} - \frac{\bar{T}}{c_p} \right]^{-1} = \frac{g}{\bar{T}} [L_d - L]^{-1}.$$

Define $A = \frac{1}{2} c h^2$. Then we have arrived at the APE equation:

$$\frac{\partial A}{\partial t} + \mathbf{v}_h \cdot \nabla_h A + \omega \frac{\partial A}{\partial p} = \alpha \omega + q_v \omega \frac{\ell_v}{c_p} \frac{R}{p} - h \omega c \frac{\ell_v}{c_p} \frac{\partial \bar{q}_v}{\partial p} + \alpha \omega \frac{ch}{c_p} + A \omega \frac{\partial \log c}{\partial p} + ch \frac{Q_R}{c_p}.$$

2.2.2. Multiscale Available Potential Energy Equation

To arrive at the multiscale moist APE equation, first take MWT on both sides of Equation 7 to get,

$$\frac{\partial \hat{h}^{\sim \varpi}}{\partial t} + \nabla \cdot (\widehat{\mathbf{v}h})^{\sim \varpi} = -\hat{\omega}^{\sim \varpi} \left[\frac{\frac{\partial \bar{q}_v \ell_v}{c_p}}{\partial p} + \frac{\partial \bar{T}}{\partial p} - \frac{\bar{\alpha}}{c_p} \right] + \frac{(\widehat{\omega \alpha})^{\sim \varpi}}{c_p} + \frac{\widehat{Q_R}^{\sim \varpi}}{c_p}. \quad (8)$$

By the definition of APE, the APE on scale window ϖ , A^{ϖ} is

$$A^{\varpi} = \frac{1}{2} c (\hat{h}^{\sim \varpi})^2.$$

Multiplication of Equation 8 by $c \hat{h}^{\sim \varpi}$ thence results in

$$\begin{aligned} \frac{\partial A^{\varpi}}{\partial t} + c \hat{h}^{\sim \varpi} \nabla \cdot (\widehat{\mathbf{v}h})^{\sim \varpi} &= -c \hat{h}^{\sim \varpi} \hat{\omega}^{\sim \varpi} \left[\frac{\frac{\partial \bar{q}_v \ell_v}{c_p}}{\partial p} + \frac{\partial \bar{T}}{\partial p} - \frac{\bar{\alpha}}{c_p} \right] + c \hat{h}^{\sim \varpi} \frac{(\widehat{\omega \alpha})^{\sim \varpi}}{c_p} + c \hat{h}^{\sim \varpi} \frac{\widehat{Q_R}^{\sim \varpi}}{c_p} \\ &= \hat{\alpha}^{\sim \varpi} \hat{\omega}^{\sim \varpi} + \frac{l_v}{c_p} \frac{R}{p} \hat{q}_v^{\sim \varpi} \hat{\omega}^{\sim \varpi} - \frac{c \ell_v}{c_p} \frac{\partial \bar{q}_v}{\partial p} \hat{h}^{\sim \varpi} \hat{\omega}^{\sim \varpi} + \frac{c}{c_p} \hat{h}^{\sim \varpi} (\widehat{\omega \alpha})^{\sim \varpi} \\ &\quad + \frac{c}{c_p} \hat{h}^{\sim \varpi} \widehat{Q_R}^{\sim \varpi}. \end{aligned}$$

Following the procedure in Liang (2016), write this as

$$\frac{\partial A^{\varpi}}{\partial t} + \nabla \cdot \mathbf{Q}_A^{\varpi} = \Gamma_A^{\varpi} + b^{\varpi} + b_{\text{vapor}}^{\varpi} + S_A^{\varpi} + F_A^{\varpi},$$

where

$$\begin{aligned} b^{\varpi} &= \hat{\alpha}^{\varpi} \hat{\omega}^{\varpi}, \\ b_{\text{vapor}}^{\varpi} &= \frac{l_v}{c_p} \frac{R}{p} \hat{q}_v^{\varpi} \hat{\omega}^{\varpi} - \frac{c \ell_v}{c_p} \frac{\partial \bar{q}_v}{\partial p} \hat{h}^{\varpi} \hat{\omega}^{\varpi}, \\ F_A^{\varpi} &= \frac{c}{c_p} \hat{h}^{\varpi} \widehat{\mathbf{Q}_R^{\varpi}}, \\ \mathbf{Q}_A^{\varpi} &= \frac{1}{2} c \hat{h}^{\varpi} \widehat{(\mathbf{v}h)^{\varpi}}, \\ \Gamma_A^{\varpi} &= -c \hat{h}^{\varpi} \nabla \cdot \widehat{(\mathbf{v}h)^{\varpi}} + \nabla \cdot \mathbf{Q}_A^{\varpi} - \frac{\partial c}{2 \partial p} \hat{h}^{\varpi} \widehat{(\omega h)^{\varpi}}, \end{aligned}$$

and $S_A^{\varpi} = S' + S''$, with

$$\begin{aligned} S' &= \frac{c}{c_p} \hat{h}^{\varpi} \widehat{(\alpha \omega)^{\varpi}}, \\ S'' &= \frac{\partial c}{2 \partial p} \hat{h}^{\varpi} \widehat{(\omega h)^{\varpi}}. \end{aligned}$$

Here S' is a small correction to $b_{\text{vapor}}^{\varpi} + b^{\varpi}$, and the second is an apparent source/sink due to the vertical profile of c . By comparison S_A^{ϖ} is generally negligible.

The transfer term can be further simplified as:

$$\begin{aligned} \Gamma_A^{\varpi} &= -c \hat{h}^{\varpi} \nabla \cdot \widehat{(\mathbf{v}h)^{\varpi}} + \frac{1}{2} \nabla \cdot \left[c \hat{h}^{\varpi} \widehat{(\mathbf{v}h)^{\varpi}} \right] - \frac{\partial c}{2 \partial p} \hat{h}^{\varpi} \widehat{(\omega h)^{\varpi}} \\ &= \frac{c}{2} \left[\widehat{(\mathbf{v}h)^{\varpi}} \cdot \nabla_h \hat{h}^{\varpi} - \hat{h}^{\varpi} \nabla_h \cdot \widehat{(\mathbf{v}h)^{\varpi}} \right] + \frac{c}{2} \left[\widehat{(\omega h)^{\varpi}} \frac{\partial \hat{h}^{\varpi}}{\partial p} - \hat{h}^{\varpi} \frac{\partial \widehat{(\omega h)^{\varpi}}}{\partial p} \right] \\ &= \frac{c}{2} \left[\widehat{(\mathbf{v}h)^{\varpi}} \cdot \nabla \hat{h}^{\varpi} - \hat{h}^{\varpi} \nabla \cdot \widehat{(\mathbf{v}h)^{\varpi}} \right] \end{aligned}$$

Following the procedure in Liang (2016), it can be proved that

$$\sum_{\varpi} \sum_n \Gamma_A^{\varpi} = 0,$$

implying that Γ_A^{ϖ} represents a process which does not generate nor destroy energy as a whole. That is to say, energy should be conserved during cross-scale interaction. This requirement seems natural but is by no means an easy task; it does not hold in traditional formalisms.

In spherical coordinates $(\lambda, \varphi, p; t)$ with the radius of Earth signified by a ,

$$\nabla \cdot \mathbf{Q}_A^{\varpi} = \frac{c}{2a \cos \varphi} \frac{\partial \left[\widehat{(\omega h)^{\varpi}} \hat{h}^{\varpi} \right]}{\partial \lambda} + \frac{c}{2a \cos \varphi} \frac{\partial \left[\widehat{(\mathbf{v}h)^{\varpi}} \hat{h}^{\varpi} \cos \varphi \right]}{\partial \varphi} + \frac{1}{2} \frac{\partial \left[c \widehat{(\omega h)^{\varpi}} \hat{h}^{\varpi} \right]}{\partial p},$$

$$\Gamma_A^\varpi = \frac{c}{2} \left[\frac{1}{a \cos \varphi} (\widehat{uh})^{\sim \varpi} \frac{\partial \widehat{h}^{\sim \varpi}}{\partial \lambda} + \frac{1}{a} (\widehat{vh})^{\sim \varpi} \frac{\partial \widehat{h}^{\sim \varpi}}{\partial \varphi} + (\widehat{\omega h})^{\sim \varpi} \frac{\partial \widehat{h}^{\sim \varpi}}{\partial p} - \frac{1}{a \cos \varphi} \widehat{h}^{\sim \varpi} \frac{\partial (\widehat{uh})^{\sim \varpi}}{\partial \lambda} - \frac{1}{a} \widehat{h}^{\sim \varpi} \frac{\partial (\widehat{vh})^{\sim \varpi} \cos \varphi}{\partial \varphi} - \widehat{h}^{\sim \varpi} \frac{\partial (\widehat{\omega h})^{\sim \varpi}}{\partial p} \right].$$

2.2.3. Multiscale APE and KE Equations for a Moist Atmosphere

For easy reference, the above moist APE equation and the KE equation from Liang (2016) are summarized as follows:

$$\begin{aligned} \frac{\partial A^\varpi}{\partial t} = & - \underbrace{\frac{1}{2} c \widehat{h}^{\sim \varpi} \nabla \cdot (\widehat{vh})^{\sim \varpi}}_{\nabla \cdot \mathbf{Q}_A^\varpi} + \underbrace{\frac{c}{2} [(\widehat{vh})^{\sim \varpi} \cdot \nabla \widehat{h}^{\sim \varpi} - \widehat{h}^{\sim \varpi} \nabla \cdot (\widehat{vh})^{\sim \varpi}]}_{\Gamma_A^\varpi} \\ & + \underbrace{\widehat{\omega}^{\sim \varpi} \widehat{\alpha}^{\sim \varpi}}_{b^\varpi} + \underbrace{\frac{l_v R}{c_p p} \widehat{q}_v^{\sim \varpi} \widehat{\omega}^{\sim \varpi} - \frac{c \ell_v}{c_p} \frac{\partial \widehat{q}_v^{\sim \varpi}}{\partial p} \widehat{h}^{\sim \varpi} \widehat{\omega}^{\sim \varpi}}_{b_{\text{vapor}}^\varpi} + \underbrace{\frac{c}{c_p} \widehat{h}^{\sim \varpi} (\widehat{\alpha \omega})^{\sim \varpi} + \frac{\partial c}{2 \partial p} \widehat{h}^{\sim \varpi} (\widehat{\omega h})^{\sim \varpi}}_{S_A^\varpi} + F_A^\varpi \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{\partial K^\varpi}{\partial t} = & \underbrace{\frac{1}{2} \{ (\widehat{\mathbf{v}v}_h)^{\sim \varpi} : \nabla \widehat{\mathbf{v}}_h^{\sim \varpi} - [\nabla \cdot (\widehat{\mathbf{v}v}_h)^{\sim \varpi}] \cdot \widehat{\mathbf{v}}_h^{\sim \varpi} \}}_{\Gamma_K^\varpi} - \underbrace{\nabla \cdot \left[\frac{1}{2} (\widehat{\mathbf{v}v}_h)^{\sim \varpi} \cdot \widehat{\mathbf{v}}_h^{\sim \varpi} \right]}_{\nabla \cdot \mathbf{Q}_K^\varpi} \\ & - \underbrace{\nabla \cdot (\widehat{\mathbf{v}}^{\sim \varpi} \widehat{\Phi}^{\sim \varpi})}_{\nabla \cdot \mathbf{Q}_P^\varpi} - \underbrace{\widehat{\omega}^{\sim \varpi} \widehat{\alpha}^{\sim \varpi}}_{b^\varpi} + F_K^\varpi \end{aligned} \quad (10)$$

where A^ϖ, K^ϖ are available potential energy and kinetic energy on window ϖ . The colon operator ($:$) in Equation 10 is defined such that, for two dyadic products $\mathbf{AB}$ and $\mathbf{CD}$, $(\mathbf{AB}) : (\mathbf{CD}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D})$. The other notations are conventional.

In the two equations, the $\mathbf{Q}$ -terms $\nabla \cdot \mathbf{Q}_A^\varpi$ ($\nabla \cdot \mathbf{Q}_K^\varpi$) and $\nabla \cdot \mathbf{Q}_P^\varpi$ are the convergence of the moist APE (KE) flux and pressure work, respectively—they represent the transport processes within the same window ϖ , the Γ -terms Γ_A^ϖ and Γ_K^ϖ are the transfers of moist APE and KE to window ϖ from other windows, b^ϖ is the rate of buoyancy conversion from moist APE to KE within window ϖ (positive if converted from APE to KE), b_{vapor}^ϖ is the APE sources/sinks related with moisture, S_A^ϖ is the apparent source/sink due to the vertical variation of static stability (usually negligible), and the F -term F_A^ϖ (F_K^ϖ) represents the diffusion (dissipation) processes. For further details regarding the derivation and interpretation of the equations, please refer to Section 2b, and the related sections in Liang (2016).

For the transfers Γ^ϖ (Γ_A^ϖ and Γ_K^ϖ), as proved by Liang (2016), they sum to zero over all windows and time steps. They bear a Lie bracket form, and satisfy the Jacobian identity, just like the Poisson bracket in theoretical physics. For this reason, this kind of transfers are called *canonical transfers* (Liang, 2016). Liang and Robinson (2007) rigorously proved that, Γ_A^ϖ and Γ_K^ϖ correspond exactly to baroclinic instability and barotropic instability, respectively. So, in terms of canonical transfer, the fully localized multiscale interactions underlying a storm can be quantitatively investigated. So far, the theory has been successfully applied to the studies of storm track (Zhao et al., 2019), atmospheric blocking (Ma & Liang, 2023), cold wave outbreak (Xu & Liang, 2020), North Atlantic Oscillation (Ma & Liang, 2025), Madden-Julian oscillation (Ma et al., 2023), Gulf of Mexico circulation (Yang et al., 2020), to name but a few.

The working line of this study consists of two steps. First, the fields are transformed and reconstructed onto different scale windows using MWT (introduced in Section 2a); the results are then analyzed in Section 5.1. Second, the scale-separated fields are used to calculate the energetics according to Equations 9 and 10, with the results presented in Section 5.2.

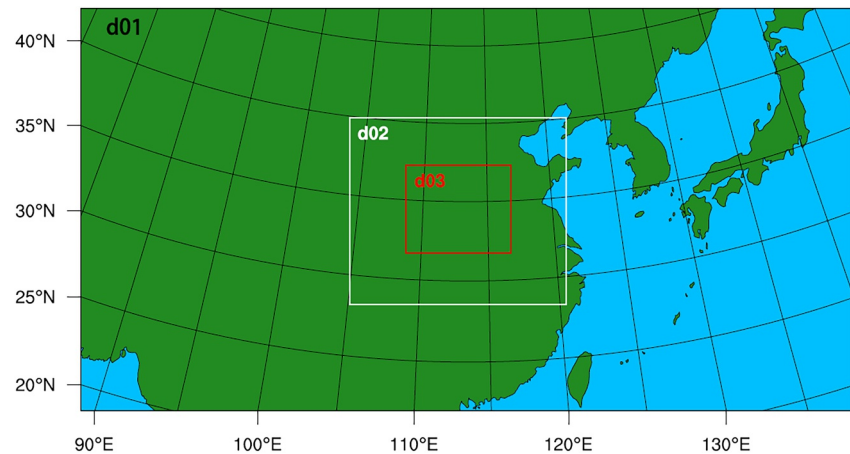


Figure 3. The grid configuration used for the WRF-ARW simulation. The three nested domains (from d01 to d03) are marked by rectangles.

3. Data

Mesoscale processes are known to play a critical role in the dynamics of extreme precipitation; therefore, a high-resolution model simulation is required to resolve these processes. This simulation will be described in detail in the next section, on which all the dynamical analyses henceforth are based. For verification, three observation/reanalysis data sets are used: the hourly rainfall data, the radar composite reflectivity data and the ERA5 reanalysis data (Hersbach et al., 2020, 2023). The hourly rainfall data collected with rain gauge is from the National Meteorological Information Center, China Meteorological Administration (2025) (<https://data.cma.cn/data/cdcdetail/dataCode/A.0012.0001.html>). The observed radar composite reflectivity data is made available by Sun et al. (2023). The ERA5 geopotential and wind fields on 850, 500, and 200 hPa are used to verify the simulated synoptic features.

4. Model Simulation and Validation

4.1. Model Setup

The model employed for this study is the widely used Weather Research and Forecasting Model, or WRF for short. The version used here is WRF-ARW, ver. 4.4.1. The model includes the Morrison double-moment microphysics scheme (Morrison et al., 2009), the rapid radiative transfer model (RRTM) (Mlawer et al., 1997), the Dudhia scheme (Dudhia, 1989) for long- and shortwave radiative flux calculations, the Mellor–Yamada–Janjic TKE (turbulent kinetic energy) scheme for the planetary boundary layer (Janjić, 1994), the Monin–Obukhov (Janjic Eta) scheme for the surface layer (Janjić, 1994), and the unified Noah land surface model (Tewari et al., 2004). The cumulus parameterization is the Kain–Fritsch cumulus parameterization scheme (Kain, 2004). More details are referred to <https://www.mmm.ucar.edu/models/wrf>.

The July 21 rainstorm over Zhengzhou is simulated in three two-way interactive, triply nested grids, whose respective horizontal resolutions are 18, 6, and 2 km. The model domains are displayed in Figure 3, where the outermost domain (d01) is centered at 34.5°N, 113.5°E, and has 301×156 mesh points, the intermediate domain (d02, centered at 34.39°N, 113.67°E) has 253×217 points, and the inner domain (d03, centered at 34.52°N, 113.48°E) has 367×307 points. By this setting, the fine structures of the meso- γ -scale processes (see Figures 10 and 12), which are the focus of this study, can be well resolved. The grids have a total of 58 sigma levels in the vertical, with the top at 20 hPa.

The Kain–Fritsch cumulus parameterization scheme (Kain, 2004) is utilized for the outer domain, but it is bypassed in the inner two domains. The model is integrated for 72 hr for all domains, starting from 0000 UTC (0800 Beijing Time) 19 July 2021, with outputs at an interval of 6 min. The initial condition and the outermost boundary condition are supplied by the interpolated data from the final operational global analysis data of the Global Forecasting System of NCEP (NCEP-FNL, <http://rda.ucar.edu/datasets/ds083-3>), with a 0.25-degree horizontal resolution and a 6-hr temporal resolution.

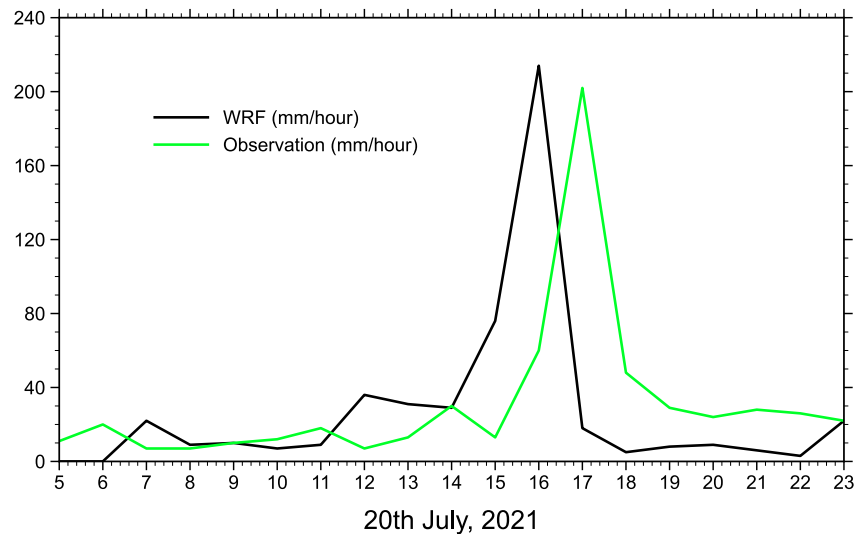


Figure 4. The hourly rainfall time series at the simulated maximum precipitation point (113.22°E , 34.23°N) (black lines) and the observed hourly rainfall time series at the Zhengzhou station (green lines) (units: mm).

4.2. The Extremely Hourly Rainfall

As we are about to reproduce the extreme hourly rainfall and then investigate the corresponding dynamics, we check the maximum hourly rainfall in the simulation. The simulation produces an hourly rainfall of 214.38 mm at (113.22°E , 34.23°N) during 1500–1600BJT, which is close to the observed amount of 201.9 mm at the Zhengzhou station. Aside from the maximum, as shown in Figure 4, the time series of the simulated precipitation is also similar to the observed time series from the rain gauge at Zhengzhou station, except for a 1-hr advance. The time discrepancy in convection development has also been found in other model simulations of this event, for example, those by Yin, Gu, Liang, et al. (2022), Yin, Gu, Yu, et al. (2022), Wei et al. (2023), etc. As this study aims to investigate the underlying dynamics, the event 1 hr earlier in the simulation is taken as the surrogate for the extreme event in the real world.

The simulated radar reflectivity at 1500 BJT is shown in Figure 5b. Compared with the observation at 1600 BJT in Figure 5a (Note that the aforementioned 1-hr advance), the simulation reasonably reproduces the observed reflectivity structure. A mesoscale convective system is seen in our simulation, consistent with the observation, in the eastern/northeastern quadrant of a mesoscale vortex, which is believed to be typical in convection-preferring regions (e.g., Wei et al., 2023). What's more, the simulated extreme hourly rainfall shows up at the southeast part of the convective system—a prominent characteristic of the observed radar reflectivity.

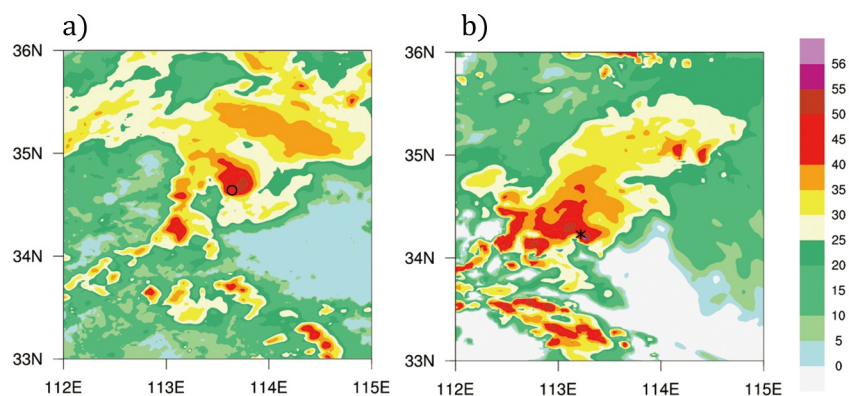
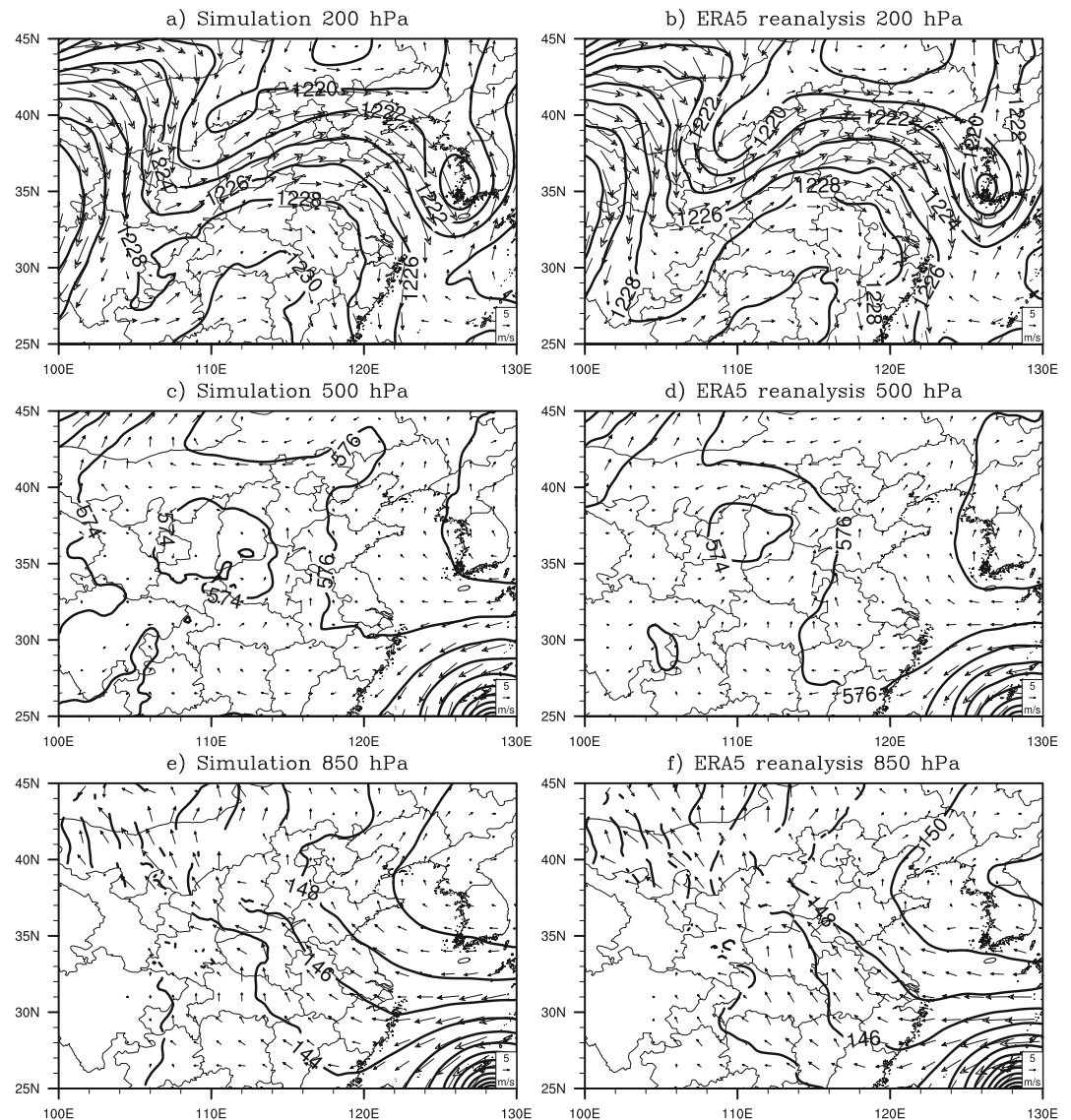


Figure 5. Observed (a) and simulated (b) radar composite reflectivity (shaded, in dBZ). The circle and the star mark the location of the observed and simulated extreme hourly rainfall.



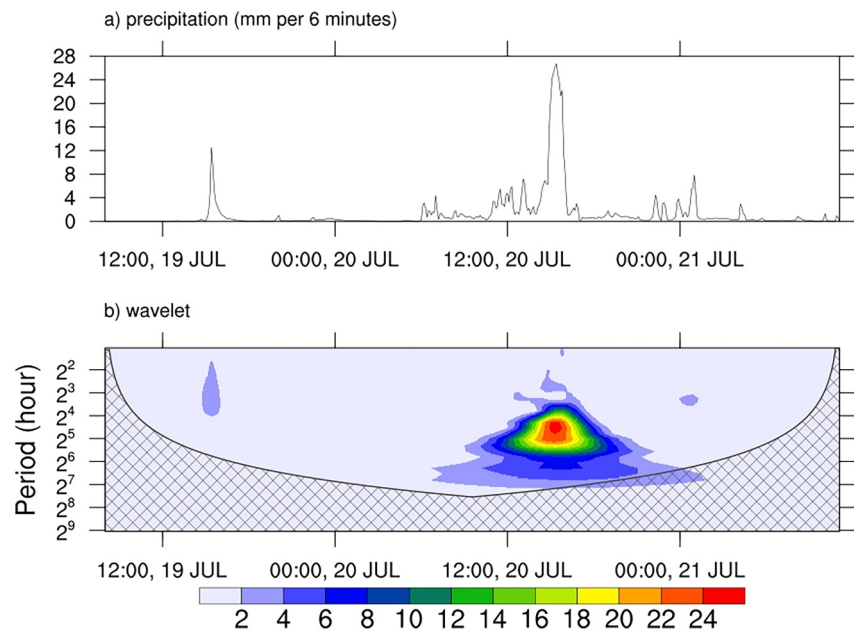


Figure 7. Time series of simulated rainfall per 6 min (top, units: mm) and its wavelet power spectrum (bottom) at the maximum precipitation point.

cyclonic vortices are found at 500 hPa both in the simulation and observation. At 850 hPa, similar cyclonic wind shears near the maximum precipitation point in the simulation and observation are clear. These facts demonstrate that our simulation successfully reproduces the observed synoptic features.

5. Results

5.1. Multiscale Reconstruction

To apply the MWT to the simulated fields, the window bounds must be first determined. Figure 7 is the wavelet power spectrum of the precipitation at the location of maximum precipitation in the simulation. Clearly, the energies on different scales have a burst near the instant when the extreme hourly precipitation takes place. By checking the distribution of energy with scale at that instant, the signals can be well separated into three scale ranges: those with periods shorter than 2^6 time steps (6.4 hr), those with periods between 2^6 (6.4 hr) and 2^8 time steps (25.6 hr), and those with periods longer than 2^8 time steps (25.6 hr). Accordingly, three scale windows are demarcated, which for convenience will be denoted as $\varpi = 0, 1, 2$, respectively. Though the separation is performed in the temporal domain, we will name them based on the spatial scale of the separated processes obtained, given that spatial and temporal scales are well correlated in the atmosphere, in order to make it easier for readers to relate the results here to others. The processes on the window $\varpi = 0$ have a relatively uniform easterly wind in the domain (Figure 8) and hence corresponding to the synoptic background; and the processes on the window $\varpi = 1, 2$ are manifested by wind convergence zones with an approximate 40 km (meso- β) and 15 km (meso- γ) spatial scale, respectively (Figures 9 and 10). Clearly, they correspond to the synoptic, meso- β -scale, and meso- γ -scale processes, and we hence will call them, respectively, the synoptic ($\varpi = 0$), meso- β -scale ($\varpi = 1$), and meso- γ -scale ($\varpi = 2$) windows. In the following the reconstructions on these windows are presented in sequel.

On the synoptic-scale window, there is a relatively uniform easterly wind in the domain (Figure 8). It is consistent with the observation in previous studies that, the southeasterlies at the northern flank of “In-fa” typhoon over the East China Sea, together with those at the southern flank of the northwest Pacific subtropical high over the Japan Sea, form a southeasterly wind corridor (e.g., Luo & Du, 2023; Su et al., 2021; Xu et al., 2022). As expected, the critical role of the low-level easterlies in transporting moisture from the ocean to Zhengzhou has been particularly emphasized for the whole heavy precipitation event in all the above-mentioned studies.

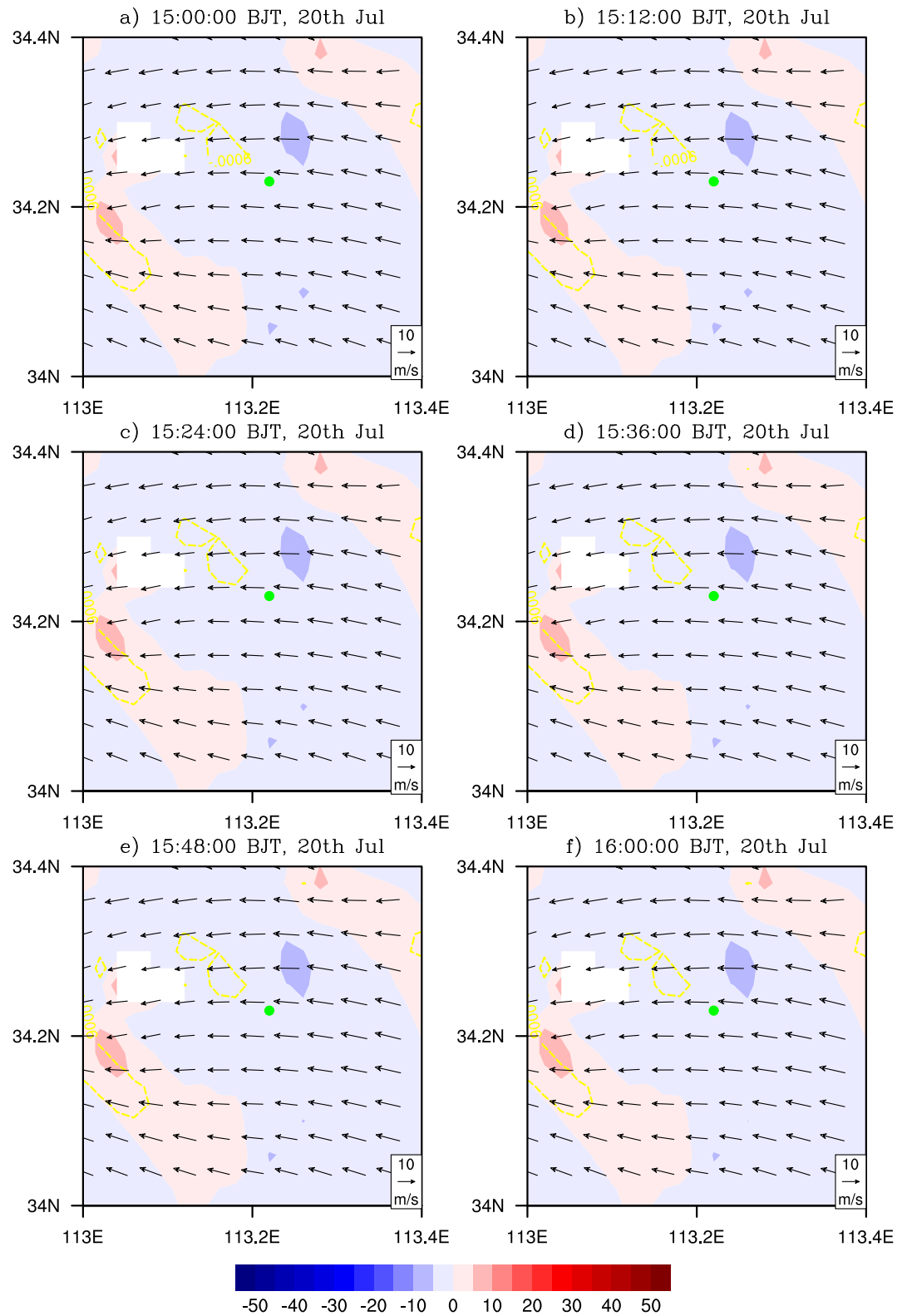


Figure 8. Reconstructed winds (vectors, in m/s), vertical velocity (shaded, in Pa/s), and divergence (contoured, in s^{-1}) on the synoptic-scale window ($\bar{\omega} = 0$) at 925 hPa.

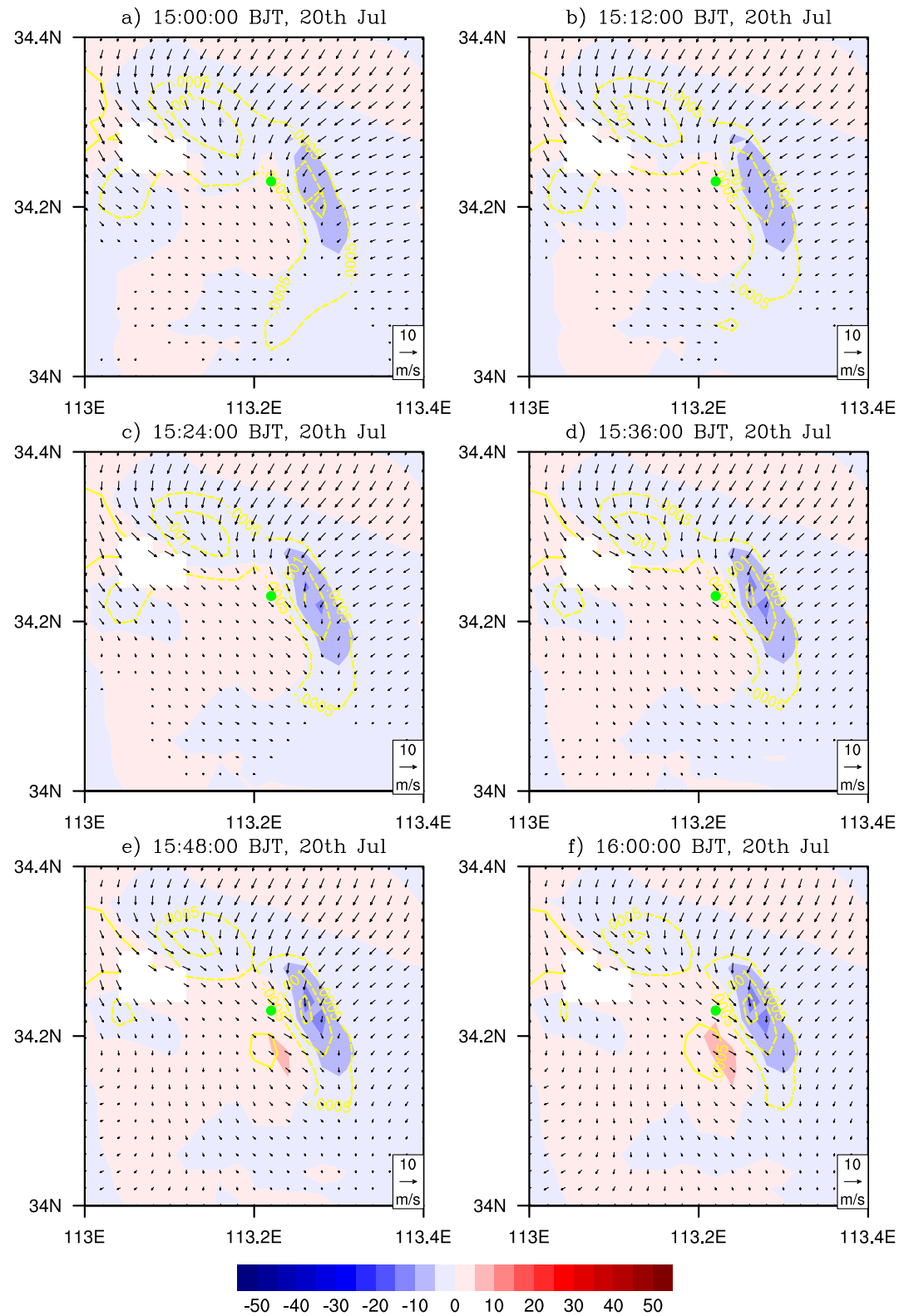


Figure 9. As in Figure 8, except for that on the meso- β -scale window ($\tau = 1$).

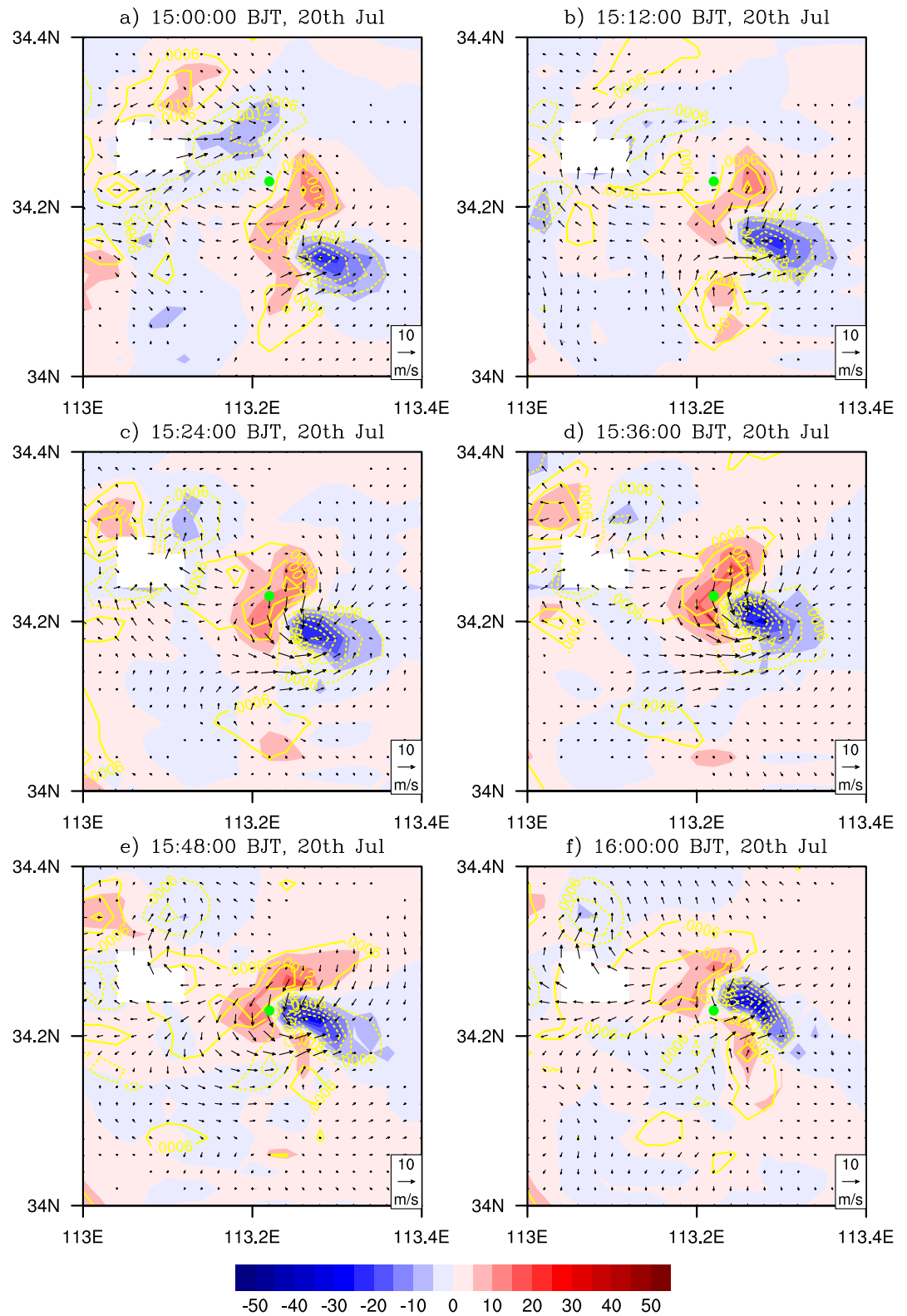


Figure 10. As in Figure 8, except for that on the meso- γ -scale window ($\varpi = 2$).

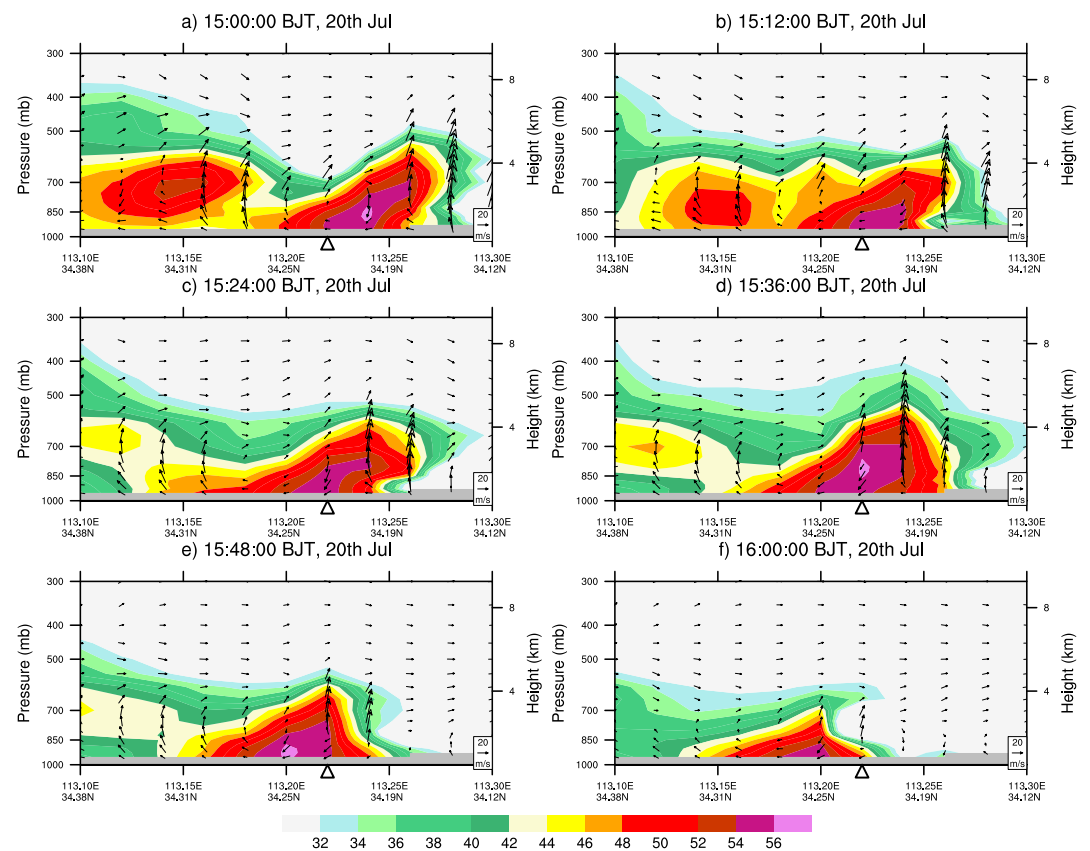


Figure 11. Vertical sectional distributions of radar composite reflectivity (shaded, in dBZ) and zonal-vertical wind (vectors, in m/s) along the line from (113.1°E, 34.38°N) to (113.3°E, 34.12°N). The triangle marks the location of the simulated maximum precipitation point.

On the meso- β -scale window, significant northerlies converge into the storm (Figure 9). Similar result has been obtained in Yin, Gu, Liang, et al. (2022), Wei et al. (2023) and Sun et al. (2023), in which the meso- β -scale winds converge at the storm and the northerly component of the wind dominates the convergence in the low layer. Wei et al. (2023) believe that the northerly component of the meso- β -scale wind balances the northward moving of the storm, explaining why the storm slows down during the extreme hourly rainfall period. Yin, Gu, Liang, et al. (2022) suggests that the arc-shape convergence of the meso- β -scale wind causes the extreme hourly rainfall directly. The role of southward wind convergence caused by the barrier jet and gravity current in intensifying the convective cell has also been emphasized by Sun et al. (2023).

By reconstruction, the obtained meso- γ -scale field retains only the fine structures of the storm while excluding the synoptic- and meso- β -scale processes, thereby facilitating a clearer understanding of its dynamics. To be specific, on the meso- γ -scale window, the most prominent feature is a weak meso- γ -scale cyclonic vortex moving northward from the southeastern side of the maximum precipitation point (Figure 10). By examining the reconstructed vortex, one can immediately see that it is accompanied by the very meso- γ -scale convective storm—it has been well separated, using MWT, from the ambient processes superimposed from other scale windows. Specifically, the updrafts and downdrafts of the storm are now clearly distinguishable—intense updrafts exist on the southeastern part of the storm, and relatively weaker downdraft on its northwestern part (Figure 10). Also observed is that the maximum rainfall (denoted by the green dot) occurs on the northwestern flank of the storm updrafts. This makes sense, as we can see from a southeast-northwest section (Figure 11), where the radar reflectivity tilts northwestward down to the surface. This means that raindrops formed in the storm drift northwestward during falling. The cause of this tilting can be illustrated by the northwest tilting updrafts (shaded) as displayed in Figure 12. It is clearly shown that the updrafts in the storm tilt northwestward slightly with height. This northwestward horizontal component of the updrafts will carry the raindrop to the northwestern side of the storm. As a result, the maximum precipitation occurs at the northwestern flank of the

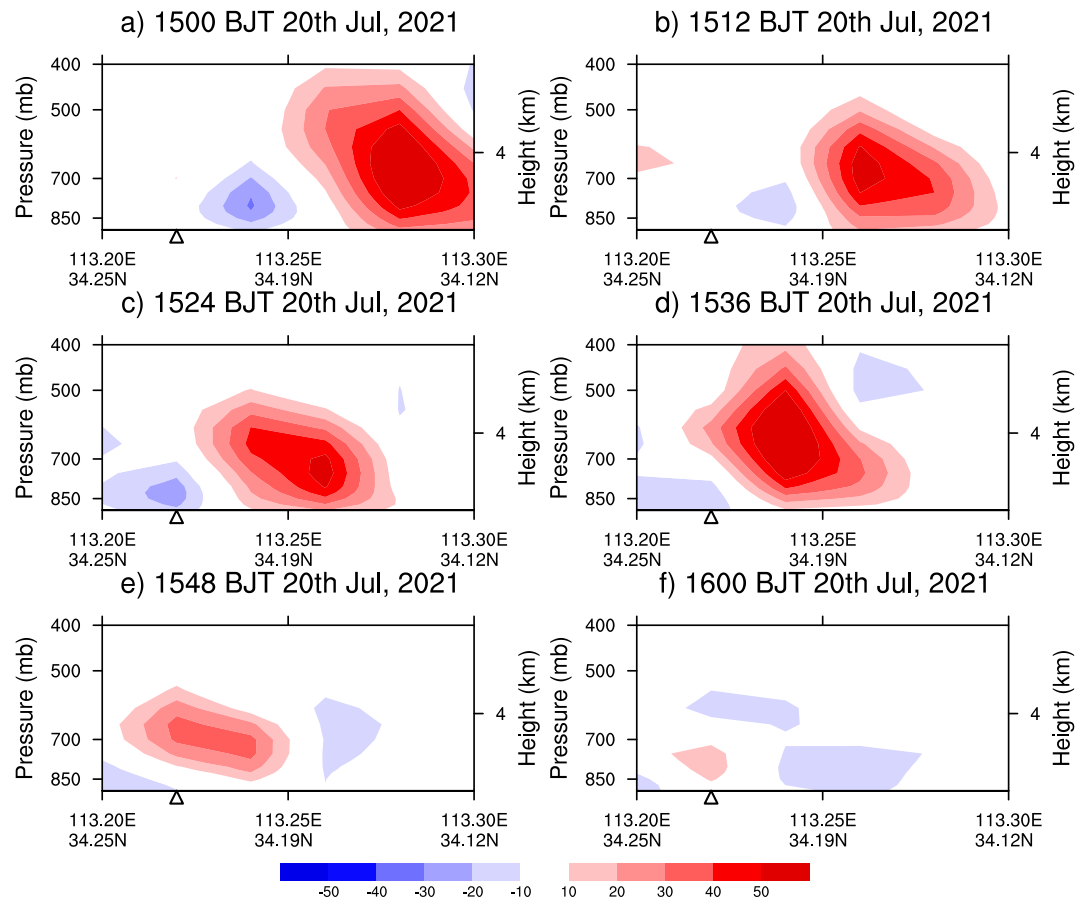


Figure 12. Vertical sectional distribution of vertical velocity (units: Pa/s) on the meso- γ -scale window along the line from (113.1°E, 34.38°N) to (113.3°E, 34.12°N). The triangle marks the location of the simulated maximum precipitation point.

updraft. Actually, the updraft tilting is a common feature in a well-developed storm; otherwise, the drag force of raindrops will slow down the updraft, leading to a weakening or even decaying storm. These results demonstrate that the meso- γ -scale processes directly drive the rainstorm. A comparison of the magnitudes of the vertical motion and divergence fields across the three windows also supports this conclusion: they are much stronger on the meso- γ -scale window than their counterparts on other windows.

As shown above, the extreme hourly rainfall is found related to a meso- γ -scale storm, consistent with previous studies (e.g., Li et al., 2025; Sun et al., 2023; Yin, Gu, Liang, et al., 2022). What is new here is that the reconstructed fields on different scale windows make it possible to qualitatively compare the relative importance of the dynamical processes on different scales. To this end, the reconstructed vertical velocity and divergence fields are shown in Figures 8–10. Notably, both fields on the meso- γ -scale window (Figure 10) are much stronger than their counterparts on the other two windows (see Figures 8 and 9), with magnitudes approximately 3–5 times larger. That is to say, the dominant dynamical process associated with the extreme hourly rainfall are the updrafts and divergences within the meso- γ -scale storm, that is, the accompanying meso- γ -scale vortex. This forms a solid basis for the forthcoming dynamic analysis.

5.2. Dynamics of the Meso- γ -Scale Convective Storm

In this section, the dynamics of the meso- γ -scale storm are investigated from the perspective of energetics. Firstly, an energy cycle underlying the storm is constructed. A physical explanation is then provided for understanding the main features of the energy cycle.

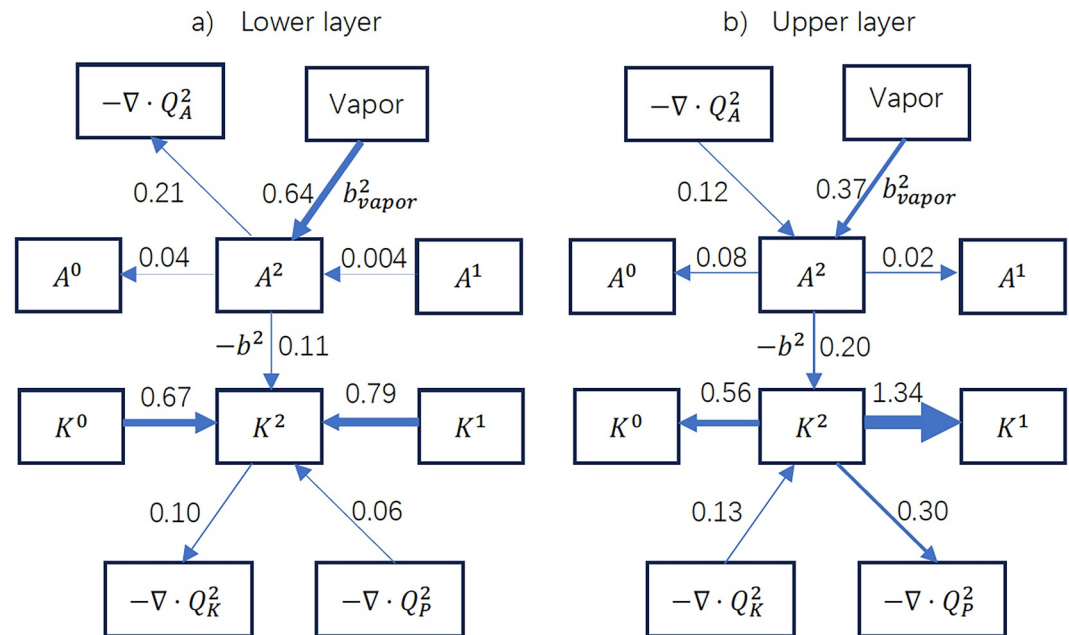


Figure 13. Energetics (units: m/s^{-3}) averaged over the meso- γ -scale convective storm region [113.2°–113.3°E, 34.1°–34.3°N] and through 925–700 hPa (a) and 700–500 hPa (b). A and K are moist available potential energy and kinetic energy, and $-\nabla \cdot Q_K$ and $-\nabla \cdot Q_P$ represent KE transport and pressure work processes. The superscripts 0, 1, and 2 refer to the synoptic, meso- β -scale and meso- γ -scale windows. The thickness of the arrows is proportional to the magnitude of the respective terms.

5.2.1. Energy Sources of the Convective Storm

According to Equations 9 and 10, a localized energy cycle is constructed, as shown in Figure 13. After examining the characteristics of the storm energetics at each level, we find that they are only significant under 500 hPa and have different scenarios in the lower and upper layers; notably, the cross-scale energy transfers beneath 700 hPa are opposite to those above it. We hence construct the Lorenz energy cycles for the layers between 925 and 700 hPa and that between 700 and 500 hPa, which are referred to as the lower and upper layer, respectively. Overall, the lower layer is a KE source while the upper layer a sink. In other words, the lower layer has a surplus while the upper layer has a deficit of KE. Part of the lower layer KE is transported to the upper layer to make up the deficit there. For the lower layer, its main KE sources are the canonical KE transfers from the synoptic and meso- β -scale windows, and buoyancy conversion. For the upper layer, its main KE sources include the buoyancy conversion and the KE transport from the lower layer. For the APE, the moisture-related processes and buoyancy conversion to KE on the meso- γ -scale window are the main APE sources and sinks of the storm, respectively, both in the lower and upper layer. The lower layer is found to transport APE to the upper layer, that is, the APE transport acts as a source and sink, respectively, for the upper and lower layer. The cross-scale canonical APE transfer is negligible. Note that the energetics obtained here are not sensitive to the area where they are averaged. We have extended the averaged area to one four times larger, and the corresponding energetics are consistent with the results here.

According to Liang and Robinson (2007), the canonical KE transfer from a lower-frequency flow to higher-frequency processes corresponds precisely to the classical barotropic instability, and the magnitude measures the rate of the perturbation growth. Hence, here barotropic instability dominates the maintenance of the storm. Similarly, the canonical APE transfer from a larger-scale circulation to smaller-scale processes corresponds precisely to the classical baroclinic instability (Liang & Robinson, 2007). As we have shown above, it is negligible. Consequently, it is reasonable to conclude that the barotropic instability of the background facilitates the maintenance of the storm, while the baroclinic instability is insignificant (two orders of magnitude smaller than its barotropic counterpart).

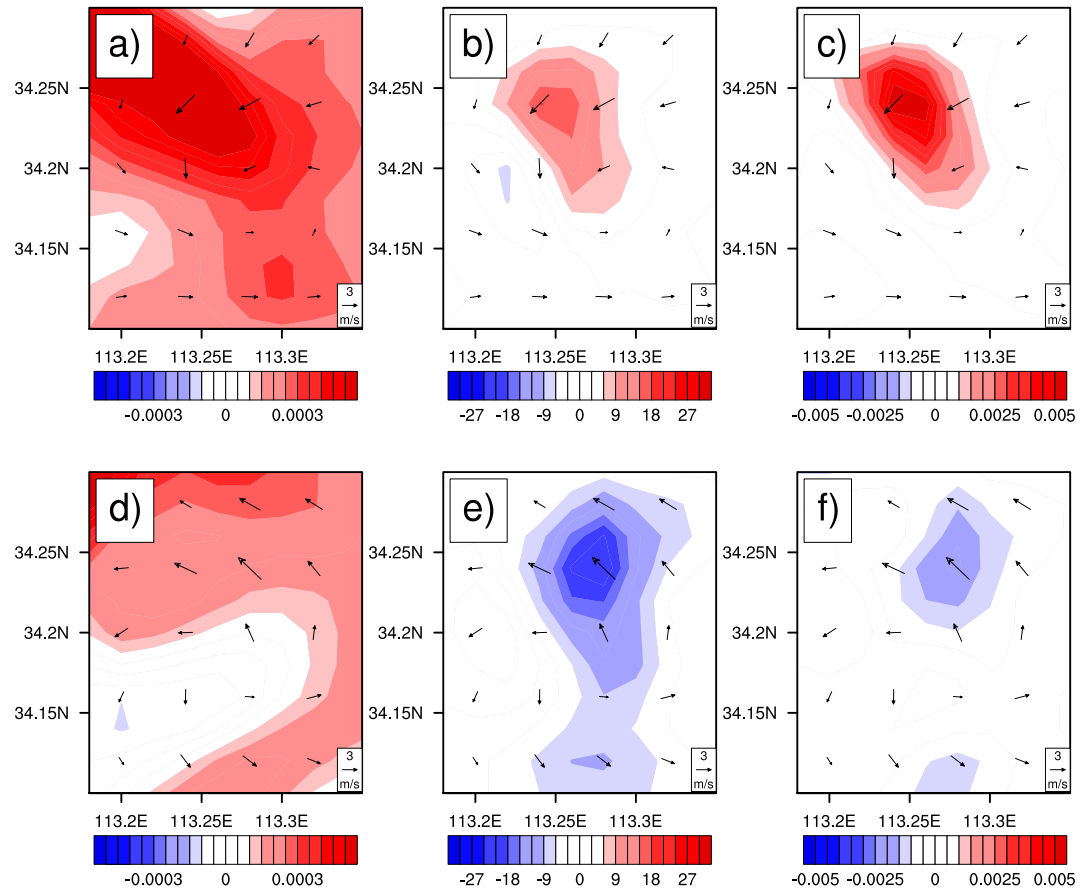


Figure 14. (a, d) The environmental vorticity due to the meridional shear of the zonal wind ($-\partial\bar{u}/\partial y$, in s^{-1}). (b, e) The product of the zonal and meridional components of the storm vortex ($\overline{u'v'}$, in m^2/s^2). (c, f) The product of the above two terms ($-1/2\overline{u'v'}\partial\bar{u}/\partial y$, in m^2/s^3). The vectors are $(\bar{u}', \bar{v}')$. All the variables are averaged over the period of 15:00–16:00 BJT, that is, the period of the extreme-rainfall hour. The values in the top (bottom) panel are averaged throughout the 925–700 hPa (700–500 hPa) layer.

5.2.2. Physical Interpretation

A main finding in the last subsection is that the convective storm extracts KE from the environment in the lower layer, but loses KE to the environment in the upper layer. A physical explanation is provided in this subsection to unravel the mechanisms behind. For the sake of simplicity and easy understanding, the canonical KE transfer within a simple Reynolds decomposition framework (note the Reynolds decomposition is the simplest MWT; see Liang and Anderson (2007)) is adopted here. As given in Liang and Robinson (2007), it is:

$$1/2[\bar{u}\nabla \cdot (\mathbf{v}'\mathbf{u}') - \overline{u'v'}(\partial\bar{u}/\partial y)] \quad (11)$$

where $\bar{u}$ is the zonal environmental wind (i.e., the wind with perturbation components removed), $\mathbf{v}' = (u', v')$ is the horizontal component of the perturbation velocity—here the horizontal wind vector of the storm vortex, and $\partial\bar{u}/\partial y$ is the meridional gradient of $\bar{u}$.

Calculation shows that the second term in the bracket in Equation 11 is dominant—two orders larger than the first term. We hence focus on the second term and analyze its two factors, that is, the meridional gradient of the environmental zonal wind term $-\partial\bar{u}/\partial y$ and the Reynolds stress component $-\overline{u'v'}$. From Figures 14a and 14d, we observe that the former doesn't change its sign from the lower layer to the upper layer and hence can not explain the opposite direction of the cross-scale KE transfer in these layers. It is the sign of $\overline{u'v'}$ (Figures 14b and 14e) that

determines the direction of the canonical KE transfer (Figures 14c and 14f). In the lower layer, the southwestward wind at the northwestern part of the storm vortex is much larger than that elsewhere, and u' and v' are positively correlated there. Consequently, a positive $\overline{u'v'}$ distribution dominates the vortex in the layer. While for the upper layer, it is the northwestward wind at the northeastern part dominates the vortex, where u' and v' are negatively correlated, leading to a dominant negative $\overline{u'v'}$ pattern in the layer.

In summary, the energy transfer paths in the upper and lower layers are controlled by the anisotropic wind field, hence the Reynolds stress, of the storm vortex there. A positive $\overline{u'v'}$ leads to a KE transfer from the larger-scale processes to the convective storm in the lower layer, while a negative $\overline{u'v'}$ leads to KE transfer from the convective storm to the larger-scale processes.

6. Conclusions

A record-breaking extreme hourly rainfall of 201.9 mm was observed in Zhengzhou, China, which caused flash floods, incurring a heavy casualty and numerous economic losses. This study investigates the dynamic mechanisms underlying this extreme hourly rainfall from a perspective of energetics. A convection-permitting simulation based on WRF-ARW is conducted to get the convective processes in the event. With the simulation as input, a functional analysis apparatus, multiscale window transform (MWT, Liang & Anderson, 2007), is then applied to reconstruct the fields onto three exclusive scale ranges, or scale windows, namely, the synoptic window, the meso- β -scale window, and the meso- γ -scale window. The MWT-based canonical transfer theory and multiscale energetics analysis (Liang, 2016) are extended to an atmosphere with moisture in this paper, which are then used to analyze the underlying multiscale processes and their interactions. A variety of new findings have been obtained, as summarized in the following.

First, the simulation has reproduced the extreme hourly rainfall event, with a maximum precipitation of 214.38 mm, close to the observed one. The time series of the precipitation is also similar to its observed counterpart (Figure 4). What's more, the simulated and observed extreme hourly rainfalls both occur at the south-eastern part of a convective system (Figure 5). These agreements tell that the underlying dynamics have been reproduced. As the spatial resolution is as high as 2 km (convection-permitting), the dynamical interactive processes from the environment to the storm are available for investigation.

In a wavelet power spectrum, the extreme event is found to be made of processes on three distinct scale ranges, which determine the bounds for three scale windows, that is, the synoptic-scale, meso- β -scale, and meso- γ -scale windows. On the synoptic window, reconstructed is a relatively uniform easterly wind in the lower layer, consistent with previous studies (e.g., Su et al., 2021; Xu et al., 2022). On the meso- β -scale window, convergence of the wind occurs at the storm, which is mainly caused by its northerly component, consistent with Yin, Gu, Liang, et al. (2022), Wei et al. (2023) and Sun et al. (2023). On the meso- γ -scale window, a slowly northwestward moving storm with a clear vortex emerges. By examining the ω reconstructions on the three scale windows, it is found that the vertical velocity associated with the meso- γ -scale storm dominates. Since an intense vertical velocity is necessary for an extreme rainfall, we can safely conclude that the storm is the critical in causing the extreme hourly rainstorm, consistent with previous studies (e.g., Sun et al., 2023; Yin, Gu, Liang, et al., 2022).

Considering the importance of the meso- γ -scale storm, its dynamics has been particularly investigated through multiscale energetics analysis. We have found that the storm exchanges KE with its environment—it extracts KE from the environment in the lower layer but loses KE to the environment in the upper layer. Physically, the anisotropy of the vortex winds embedded in the meso- γ -scale storm is the key accounting for the paths in the lower and upper layers. It not only helps to extract energy to fuel the vortex in the lower layer, but also accounts for the opposite canonical KE transforms in the upper and lower layers. Since anisotropy may be a common feature for meso- γ -scale vortices (e.g., Dahl, 2017), the energy exchange between a barotropic convective storm and its ambient environment deserves more attention. Also note that the moisture-related processes represented by the b_{vapor}^2 term are comparable to the KE transfers among scales, especially when the contributions from the upper and lower layers are integrated (Figure 14). These issues, among others, will be investigated in future studies.

Appendix A

A glossary of notations

- ∇ : 3D gradient operator, $\nabla = \mathbf{e}_\lambda \frac{\partial}{\partial \lambda} + \mathbf{e}_\varphi \frac{\partial}{\partial \varphi} + \mathbf{e}_p \frac{\partial}{\partial p}$
- ∇_h : horizontal gradient operator (horizontal component of ∇)
- $\mathbf{v}$: velocity $\mathbf{v} = (u, v, \omega)$, $\omega = \frac{dp}{dt}$
- $\mathbf{v}_h$ = (u, v)
- $\hat{\pi}^{\sim \varpi}$: MWT of some property π at step n on window ϖ ; dependence on n in suppressed when no confusion arises
- $\bar{T} = \bar{T}(p)$: mean temperature profile (averaged over the p -plane and time)
- T : departure from $\bar{T}$
- α : specific volume
- Φ : geopotential function
- R : specific gas constant (in $\text{J} \cdot \text{kg}^{-1} \text{K}^{-1}$)
- $c_p = 1005$: specific heat capacity of air (in $\text{J} \cdot \text{kg}^{-1} \text{K}^{-1}$) for isobaric processes
- L : Lapse rate ($L = -\frac{\partial \bar{T}}{\partial z}$)
- L_d : Lapse rate of dry air ($L_d = g/c_p$)
- a : radius of the Earth
- (λ, φ, p) : the isobaric spherical coordinates
- g : acceleration due to gravity
- Q : flux
- Γ : canonical transfer
- A : moist available potential energy
- K : kinetic energy
- b : buoyancy conversion rate

Conflict of Interest

The authors declare no conflicts of interest relevant to this study.

Availability Statement

The hourly rainfall data are from <https://data.cma.cn/data/cdcdetail/dataCode/A.0012.0001.html>, and the observed radar composite reflectivity data are from Sun et al. (2023). The ERA5 data sets used in this study are available at <https://cds.climate.copernicus.eu/datasets/reanalysis-era5-pressure-levels?tab=overview>.

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